

SPM BULLETIN

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1. EDITOR'S NOTE

Good news in brief:

- (1) The abstracts for the recent *Kielce Conference on Set-Theoretic Topology* are available online at

<http://www.pu.kielce.pl/topoconf/skrzynia/abstrakty.pdf>

and at

<http://atlas-conferences.com/cgi-bin/abstract/catc-01>

Some of the presentations of this beautiful conference should be available soon in

<http://www.pu.kielce.pl/topoconf/prezentacje.html>

- (2) Some pictures from the recent TOPOSYM and Kielce conferences are now available in Marcin Kysiak's webpage

<http://www.mimuw.edu.pl/~mkysiak>

(follow the links at the bottom of this page).

- (3) July 14-19, 2007: The *Logic Colloquium 2007* (ASL European Summer Meeting) will take place in Wrocław (Poland), one of the world's centers for SPM related mathematics. It is likely that some of the talks there will be related to SPM.

Contributions to the next issue are, as always, welcome.

Boaz Tsaban, boaz.tsaban@weizmann.ac.il

<http://www.cs.biu.ac.il/~tsaban>

2. RESEARCH ANNOUNCEMENTS

2.1. A surprising covering of the real line. We construct an increasing sequence of Borel subsets of \mathbb{R} , such that their union is \mathbb{R} , but \mathbb{R} cannot be covered with countably many translations of one set. The proof uses a random method.

<http://www.ams.org/journal-getitem?pii=S0002-9939-06-08371-7>

Gábor Kun

2.2. Unions of chains in dyadic compact spaces and topological groups. The following problem is considered: If a topological group G is the union of an increasing chain of subspaces and certain cardinal invariants of the subspaces are known, what can be said about G ? We prove that if G is locally compact and every subspace in the chain has countable pseudocharacter or tightness, then G is metrizable. We also prove a similar assertion for σ -compact and totally bounded groups represented as the union of first countable subspaces, when the length of the chain is a regular cardinal greater than \aleph_1 . Finally, we show that these results are not valid in general, not even for compact spaces.

[http://dx.doi.org/10.1016/S0166-8641\(01\)00106-7](http://dx.doi.org/10.1016/S0166-8641(01)00106-7)

Mikhail G. Tkachenko and Yolanda Torres Falcón

2.3. On the Pytkeev property in spaces of continuous functions. Answering a question of Sakai, we show that the minimal cardinality of a set of reals X such that $C_p(X)$ does not have the Pytkeev property is equal to the pseudo-intersection number \mathfrak{p} . Our approach leads to a natural characterization of the Pytkeev property of $C_p(X)$ by means of a covering property of X , and to a similar result for the

Reznichenko property of $C_p(X)$. We also give a new result of Miller: If $C_p(X)$ has the Pytkeev property, then X has strong measure zero. (To appear in *Proceedings of the AMS*.)

<http://arxiv.org/math/0606270>

Petr Simon and Boaz Tsaban

2.4. Selection principles related to α_i -properties. We investigate selection principles which are motivated by Arhangel'skii's α_i -properties, $i = 1, 2, 3, 4$, and their relations with classical selection principles. It will be shown that they are closely related to the selection principle S_1 and often are equivalent to it.

<http://arxiv.org/math/math.GN/0608107>

Ljubiša D.R. Kočinac

2.5. On the Kocinac α_i properties. The Kocinac α_i properties, $i = 1, 2, 3, 4$, are generalizations of Arkhangel'skii's α_i local properties. We give a complete classification of these properties when applied to the standard families of open covers of topological spaces or to the standard families of open covers of topological groups. One of the latter properties characterizes totally bounded groups. We also answer a question of Kocinac.

<http://arxiv.org/math/0607592>

Boaz Tsaban

2.6. A new selection principle. Motivated by a recent result of Sakai, we define a new selection operator for covers of topological spaces, inducing new selection hypotheses. We initiate a systematic study of the new hypotheses. Some intriguing problems remain open.

<http://arxiv.org/math/0606285>

Boaz Tsaban

2.7. First Countable Continua and Proper Forcing. Assuming the Continuum Hypothesis, there is a compact first countable connected space of weight \aleph_1 with no totally disconnected perfect subsets. Each such space, however, may be destroyed by some proper forcing order which does not add reals.

<http://arxiv.org/math/math.GN/0608035>

Joan E. Hart and Kenneth Kunen

2.8. The convergence space of minimal usco mappings. A convergence structure generalizing the order convergence structure on the set of Hausdorff continuous interval functions is defined on the set of minimal usco maps. The properties of the obtained convergence space are investigated and essential links with the pointwise convergence and the order convergence are revealed. The convergence structure can be extended to a uniform convergence structure so that the convergence space is complete. The important issue of the denseness of the subset of all continuous functions is also addressed.

<http://arxiv.org/math/math.GN/0608086>

R Anguelov, O. F.K. Kalenda

2.9. D -forced spaces: a new approach to resolvability. We introduce a ZFC method that enables us to build spaces (in fact special dense subspaces of certain Cantor cubes) in which we have “full control” over all dense subsets. Using this method we are able to construct, in ZFC, for each uncountable regular cardinal λ a 0-dimensional T_2 , hence Tychonov, space which is μ -resolvable for all $\mu < \lambda$ but not λ -resolvable. This yields the final (negative) solution of a celebrated problem of Ceder and Pearson raised in 1967: Are ω -resolvable spaces maximally resolvable? This method enables us to solve several other open problems concerning resolvability as well.

The paper appeared in *Top. Appl.* **153** (2006), 1800–1824.

<http://arxiv.org/math/math.GN/0609090>

Istvan Juhasz, Lajos Soukup, and Zoltan Szentmiklossy

2.10. Resolvability of spaces having small spread or extent. In a recent paper O. Pavlov proved the following two interesting resolvability results:

- (1) If a space X satisfies $\Delta(X) > \text{ps}(X)$ then X is maximally resolvable.
- (2) If a T_3 -space X satisfies $\Delta(X) > \text{pe}(X)$ then X is ω -resolvable.

Here $\text{ps}(X)$ ($\text{pe}(X)$) denotes the smallest successor cardinal such that X has no discrete (closed discrete) subset of that size and $\Delta(X)$ is the smallest cardinality of a non-empty open set in X . In this note we improve (1) by showing that $\Delta(X) > \text{ps}(X)$ can be relaxed to $\Delta(X) \geq \text{ps}(X)$. In particular, if X is a space of countable spread with $\Delta(X) > \omega$ then X is maximally resolvable. The question if an analogous improvement of (2) is valid remains open, but we present a proof of (2) that is simpler than Pavlov's.

<http://arxiv.org/math/math.GN/0609091>

Istvan Juhasz, Lajos Soukup, and Zoltan Szentmiklossy

2.11. Resolvability and monotone normality. A space X is said to be κ -resolvable (resp. almost κ -resolvable) if it contains κ dense sets that are pairwise disjoint (resp. almost disjoint over the ideal of nowhere dense subsets). X is maximally resolvable iff it is $\Delta(X)$ -resolvable, where $\Delta(X) = \min\{|G| : G \neq \emptyset \text{ open}\}$. We show that every crowded monotonically normal (in short: MN) space is ω -resolvable and almost μ -resolvable, where $\mu = \min\{2^\omega, \omega_2\}$. On the other hand, if κ is a measurable cardinal then there is a MN space X with $\Delta(X) = \kappa$ such that no subspace of X is ω_1 -resolvable. Any MN space of cardinality $< \aleph_\omega$ is maximally resolvable. But from a supercompact cardinal we obtain the consistency of the existence of a MN space X with $|X| = \Delta(X) = \aleph_\omega$ such that no subspace of X is ω_2 -resolvable.

<http://arxiv.org/math/math.GN/0609092>

Istvan Juhasz, Lajos Soukup, and Zoltan Szentmiklossy

2.12. Isomorphism of Borel full groups. Suppose that G and H are Polish groups which act in a Borel fashion on Polish spaces X and Y . Let E_G^X and E_H^Y denote the corresponding orbit equivalence relations, and $[G]$ and $[H]$ the corresponding Borel full groups. Modulo the obvious counterexamples, we show that $[G] \cong [H]$ iff E_G^X is Borel isomorphic to E_H^Y .

<http://www.ams.org/journal-getitem?pii=S0002-9939-06-08542-X>

Benjamin D. Miller; Christian Rosendal

2.13. A Poset Hierarchy. This article extends a paper of Abraham and Bonnet which generalised the famous Hausdorff characterisation of the class of scattered linear orders. Abraham and Bonnet gave a poset hierarchy that characterised the class of scattered posets which do not have infinite antichains (abbreviated FAC for finite antichain condition). An antichain here is taken in the sense of incomparability. We define a larger poset hierarchy than that of Abraham and Bonnet, to include a broader class of “scattered” posets that we call κ -scattered. These posets cannot embed any order such that for every two subsets of size $< \kappa$, one being strictly less than the other, there is an element in between. If a linear order has this property and has size κ we call this set $\mathbb{Q}(\kappa)$. Such a set only exists when $\kappa^{<\kappa} = \kappa$. Partial orders with the property that for every $a < b$ the set $\{x : a < x < b\}$ has size $\geq \kappa$ are called weakly κ -dense, and partial orders that do not have a weakly κ -dense subset are called strongly κ -scattered. We prove that our hierarchy includes all strongly κ -scattered FAC posets, and that the hierarchy is included in the class of all FAC κ -scattered posets. In addition, we prove that our hierarchy is in fact the closure of the class of all κ -well-founded linear orders under inversions, lexicographic sums and FAC weakenings. For $\kappa = \aleph_0$ our hierarchy agrees with the one from the Abraham-Bonnet theorem.

Central European Journal of Mathematics, vol. 4, no. 2, (2006), 225–241.

<http://arxiv.org/math/math.L0/0608642>

M. Džamonja and K. Thompson

2.14. Infinite asymptotic games. We study infinite asymptotic games in Banach spaces with an F.D.D. and prove that analytic games are determined by characterising precisely the conditions for the players to have winning strategies. These results are applied to characterise spaces embeddable into ℓ_p sums of finite dimensional spaces, extending results of Odell and Schlumprecht, and to study various notions of homogeneity of bases and Banach spaces. These results are related to questions of rapidity of subsequence extraction from normalised weakly null sequences.

<http://arxiv.org/math/math.FA/0608616>

Christian Rosendal

2.15. Elementary submodels and separable monotonically normal compacta. In this note, we use elementary submodels to prove that a separable monotonically

normal compactum can be mapped on a separable metric space via a continuous function whose fibers have cardinality at most 2.

<http://arxiv.org/math/math.GN/0608376>

Todd Eisworth

2.16. An application of CAT. We comment on a question of Justin Moore on colorings of pairs of nodes in an Aronszajn tree and solve an instance of it.

<http://arxiv.org/math/math.LO/0608382>

Mirna Džamonja and Jean Larson

2.17. A general Stone representation theorem. This note contains a Stone-style representation theorem for compact Hausdorff spaces.

<http://arxiv.org/math/math.LO/0608384>

Mirna Džamonja

2.18. Measure Recognition Problem. This is an article on the example of the Measure Recognition Problem (MRP). The article highlights the phenomenon of the utility of a multidisciplinary mathematical approach to a single mathematical problem, in particular the value of a set-theoretic analysis. MRP asks if for a given Boolean algebra \mathfrak{B} and a property Φ of measures one can recognize by purely combinatorial means if \mathfrak{B} supports a strictly positive measure with property Φ . The most famous instance of this problem is MRP(countable additivity), and in the first part of the article we survey the known results on this and some other problems. We show how these results naturally lead to asking about two other specific instances of the problem MRP, namely MRP(nonatomic) and MRP(separable). Then we show how our recent work Džamonja and Plebanek (2006) gives an easy solution to the former of these problems, and gives some partial information about the latter. The long term goal of this line of research is to obtain a structure theory of Boolean algebras that support a finitely additive strictly positive measure, along the lines of Maharam theorem which gives such a structure theorem for measure algebras.

<http://arxiv.org/math/math.LO/0608336>

Mirna Džamonja

2.19. Cardinal invariants for C -cross topologies. C -cross topologies are introduced. Modifications of the Kuratowski-Ulam Theorem are considered. Cardinal invariants add , cof , cov , and non with respect to meager or nowhere dense subsets are compared. Remarks on invariants $\text{cof}(nwd_Y)$ are mentioned for dense subspaces Y of X .

<http://arxiv.org/math/math.GN/0609351>

Andrzej Kucharski and Szymon Plewik

3. PROBLEM OF THE ISSUE

In our work (in progress) on Ramsey theory of open covers, we have encountered the following natural problem, to which we do not see the answer.

Problem 3.1. *Assume that $X \subseteq \mathbb{R}$ is Hurewicz and all finite powers of X are Menger. Are all finite powers of X Hurewicz?*

In modern notation [1], Problem 3.1 can be stated as follows.

Problem 3.2. *Is $\mathcal{U}_{fin}(\mathcal{O}, \Gamma) \cap \mathcal{S}_{fin}(\Omega, \Omega) = \mathcal{S}_{fin}(\Omega, \Omega^{gp})$?*

$\mathcal{U}_{fin}(\mathcal{O}, \Gamma) = \mathcal{S}_{fin}(\Omega, \Lambda^{gp})$ [1], which in turn is equivalent to $\binom{\Lambda}{\Lambda^{gp}}$ [2] (see also [3]). Thus, another way to state this is:

Problem 3.3. *Is it true that, for $X \subseteq \mathbb{R}$ satisfying $\mathcal{S}_{fin}(\Omega, \Omega)$, $\binom{\Lambda}{\Lambda^{gp}} = \binom{\Omega}{\Omega^{gp}}$?*

A negative answer under the Continuum Hypothesis is what we would like to have, so here is a variant:

Problem 3.4 (CH). *Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?*

Nadav Samet and Boaz Tsaban

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- [2] B. Tsaban, *The Hurewicz covering property and slaloms in the Baire space*, Fundamenta Mathematicae **181** (2004), 273–280.
- [3] L. Zdomskyy, *A semifilter approach to selection principles*, Commentationes Mathematicae Universitatis Carolinae **46** (2005), 525–540.

4. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. *Is $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\Gamma}\right)$?*

Issue 2. *Is $\mathcal{U}_{fin}(\Gamma, \Omega) = \mathcal{S}_{fin}(\Gamma, \Omega)$? And if not, does $\mathcal{U}_{fin}(\Gamma, \Gamma)$ imply $\mathcal{S}_{fin}(\Gamma, \Omega)$?*

Issue 4. *Does $\mathcal{S}_1(\Omega, \Gamma)$ imply $\mathcal{U}_{fin}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $\mathcal{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$?*

Issue 8. *Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \mathcal{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?*

Issue 9 (CH). *Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?*

Issue 10. *Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)*

Issue 11. *Does $\mathcal{S}_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?*

Issue 12. *Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?*

Issue 14. *Does there exist (in ZFC) a set of reals X of cardinality \mathfrak{d} such that all finite powers of X have Menger's property $\mathcal{S}_{fin}(\mathcal{O}, \mathcal{O})$?*

Issue 15. *Can a Borel non- σ -compact group be generated by a Hurewicz subspace?*

Issue 16 (MA). *Is there an uncountable $X \subseteq \mathbb{R}$ satisfying $\mathcal{S}_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?*

Issue 17 (CH). *Is there a totally imperfect X satisfying $\mathcal{U}_{fin}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^\mathbb{N}$?*

Issue 18 (CH). *Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?*

Previous issues. The previous issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, at <http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22>

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in L^AT_EX. The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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